DRAFT Search for the Decay $K^+ \to \pi^+ \gamma \gamma$ in the π^+ Momentum Region $P > 213~{\rm MeV}/c$

V.V. Anisimovsky, A.V. Artamonov, B. Bassalleck, B. Bhuyan, E.W. Blackmore, 5 D.A. Bryman, ⁶ S. Chen, ⁵ I-H. Chiang, ⁴ I.-A. Christidi, ⁷ P.S. Cooper, ⁸ M.V. Diwan, ⁴ J.S. Frank, T. Fujiwara, J. Hu, A.P. Ivashkin, D.E. Jaffe, S. Kabe, 10 S.H. Kettell, M.M. Khabibullin, A.N. Khotjantsev, P. Kitching, M. Kobayashi, O T.K. Komatsubara, ¹⁰ A. Konaka, ⁵ A.P. Kozhevnikov, ² Yu.G. Kudenko, ¹ A. Kushnirenko, ^{8,†} L.G. Landsberg, ² B. Lewis, ³ K.K. Li, ⁴ L.S. Littenberg, ⁴ J.A. Macdonald,^{5,‡} J. Mildenberger,⁵ O.V. Mineev,¹ M. Miyajima,¹² K. Mizouchi,⁹ V.A. Mukhin,² N. Muramatsu,¹³ T. Nakano,¹³ M. Nomachi,¹⁴ T. Nomura,⁹ T. Numao,⁵ V.F. Obraztsov,² K. Omata,¹⁰ D.I. Patalakha,² S.V. Petrenko,² R. Poutissou, ⁵ E.J. Ramberg, ⁸ G. Redlinger, ⁴ T. Sato, ¹⁰ T. Sekiguchi, ¹⁰ T. Shinkawa, ¹⁵ R.C. Strand, ⁴ S. Sugimoto, ¹⁰ Y. Tamagawa, ¹² R. Tschirhart, ⁸ T. Tsunemi, ¹⁰ D.V. Vavilov, B. Viren, N.V. Yershov, Y. Yoshimura, and T. Yoshioka of the control of the contro ¹Institute for Nuclear Research RAS, 60 October Revolution Pr. 7a, 117312 Moscow, Russia ²Institute for High Energy Physics, Protvino, Moscow Region, 142 280, Russia ³Department of Physics and Astronomy, University of New Mexico, Albuquerque, NM 87131 ⁴Brookhaven National Laboratory, Upton, NY 11973 ⁵TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, Canada V6T 2A3 ⁶Department of Physics and Astronomy, University of British Columbia, Vancouver, British Columbia, Canada V6T 1Z1 ⁷Department of Physics and Astronomy, Stony Brook University, Stony Brook, NY 11794

⁸Fermi National Accelerator Laboratory, Batavia, IL 60510

⁹Department of Physics, Kyoto University, Sakyo-ku, Kyoto 606-8502, Japan

¹⁰High Energy Accelerator Research Organization (KEK),

Oho, Tsukuba, Ibaraki 305-0801, Japan

¹¹Centre for Subatomic Research, University of Alberta, Edmonton, Canada T6G 2N5

¹²Department of Applied Physics, Fukui University,

3-9-1 Bunkyo, Fukui, Fukui 910-8507, Japan

¹³Research Center for Nuclear Physics, Osaka University,

10-1 Mihogaoka, Ibaraki, Osaka 567-0047, Japan

¹⁴Laboratory of Nuclear Studies, Osaka University,

1-1 Machikaneyama, Toyonaka, Osaka 560-0043, Japan

¹⁵Department of Applied Physics, National Defense Academy,

Yokosuka, Kanagawa 239-8686, Japan

(Dated: DRAFT01-March 23, 2005)

Abstract

We have performed a search for the decay $K^+ \to \pi^+ \gamma \gamma$ in the kinematic region with π^+ momentum close to the end point. No events were observed, and a 90% confidence-level upper limit on the partial branching ratio $\mathcal{B}(K^+ \to \pi^+ \gamma \gamma, \, P > 213 \,\mathrm{MeV/c})$ is determined to be 9.1×10^{-9} under the assumption of the chiral perturbation theory including next-to-leading order corrections. The same data were used to set an upper limit on the $K^+ \to \pi^+ \gamma$ branching ratio to be 2.5×10^{-9} . We also report improved limits on the rates of $K^+ \to \pi^+ X^0$ and $X^0 \to \gamma \gamma$, where X^0 is a hypothetical short-lived neutral particle.

PACS numbers: 13.20.Eb, 12.15.Hh, 14.80.Mz to be confirmed

We report the results of a new search for the rare decay $K^+ \to \pi^+ \gamma \gamma$ in the π^+ momentum region P > 213 MeV/c from the E949 experiment [1] at the Alternating Gradient Synchrotron (AGS) of Brookhaven National Laboratory (BNL). The first observation of the decay in the π^+ momentum from 100 to 180 MeV/c was reported [2] by the BNL-E787 experiment with the corresponding partial branching ratio $\mathcal{B}(K^+ \to \pi^+ \gamma \gamma, 100 \text{ MeV/}c < P < 180 \text{ MeV/}c)$ of $[6.0 \pm 1.5(stat) \pm 0.7(syst)] \times 10^{-7}$. In the π^+ momentum region greater than 215 MeV/c, no $K^+ \to \pi^+ \gamma \gamma$ decay was observed.

In an effective-field approach to low energy QCD called chiral perturbation theory (ChPT) [3], there is no tree-level $O(p^2)$ contribution to $K^+ \to \pi^+ \gamma \gamma$ and to the neutral counterpart $K_L^0 \to \pi^0 \gamma \gamma$; the leading contributions start at $O(p^4)$ [4]. For $K^+ \to \pi^+ \gamma \gamma$, both the branching ratio and the π^+ spectrum shape at $O(p^4)$ are sensitive to one undetermined coupling-constant, \hat{c} . The results from E787 were consistent with ChPT at $O(p^4)$ with $\hat{c} = 1.6 \pm 0.6$. In the next-to-leading order at $O(p^6)$, one-loop "unitarity" corrections are deduced from an empirical fit of the decay amplitude of $K^+ \to \pi^+ \pi^+ \pi^-$ and contain the same constant \hat{c} , while the contribution of vector-meson exchange is expected to be negligible compared to the unitarity corrections [5]. The best fit to the measured π^+ spectrum was obtained for $\hat{c} = 1.8 \pm 0.6$ with the unitarity corrections; the E787 data support the ChPT including the corrections but are not conclusive. For $K_L^0 \to \pi^0 \gamma \gamma$, the amplitude at $O(p^4)$ is determined but the measured branching ratio, $(1.41 \pm 0.12) \times 10^{-6}$ [6], is twice as large as the prediction. The vector meson contribution (parametrized by an effective coupling constant a_v) is considered to be important to this decay [7].

One of the consequences of the unitarity corrections is a nonzero amplitude at the kinematic region close to the end point $P=227~{\rm MeV}/c$ (the invariant mass of two photons $m_{\gamma\gamma}=0~{\rm MeV}/c^2$), as shown in Fig. 1. The partial branching ratio $\mathcal{B}(K^+\to\pi^+\gamma\gamma,\,P>213~{\rm MeV}/c)$ is predicted to be $6.10^{+0.16}_{-0.12}\times10^{-9}$ for $\hat{c}=1.8\pm0.6$ with the unitarity corrections and $0.49^{+0.23}_{-0.18}\times10^{-9}$ for $\hat{c}=1.6\pm0.6$ without the corrections. The former is one order of magnitude larger than the latter, and is within reach of E949. The same kinematic region in $K_L^0\to\pi^0\gamma\gamma$ is known to be crucial to understand the CP-conserving component to the $K_L^0\to\pi^0e^+e$ decay, but experimental results on a_v [8, 9] are inconsistent and their theoretical interpretations are controversial [10].

E949 is primarily designed to measure the rare decay $K^+ \to \pi^+ \nu \bar{\nu}$ [11]. The AGS delivered kaons of 710 MeV/c to the experiment at a rate of 9×10^6 per 2.2-s spill. Kaons,

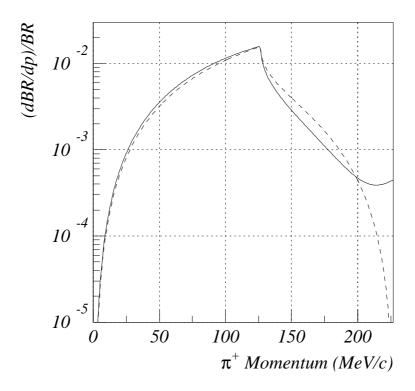


FIG. 1: Predictions for the π^+ momentum for $\hat{c} = 1.8$ with the unitarity corrections (solid line) and for $\hat{c} = 1.6$ without the corrections (dashed line), respectively, in the logarithmic scale.

detected and identified by Cerenkov, tracking, and energy-loss counters, were slowed by a Be degrader, and came to rest and decayed in a scintillating-fiber target. Fig. 2 shows a diagram of the apparatus. Measurements of charged decay products were made using the target, a central drift chamber, and a cylindrical range stack (RS) composed of 17 layers of 2-cm thick plastic scintillator with two embedded layers of tracking chambers. The pion from the $K^+ \to \pi^+ \gamma \gamma$ decay was identified by comparing momentum, range (R), and kinetic-energy (E) measurements, and by observation of the $\pi^+ \to \mu^+ \to e^+$ decay sequence at rest in the RS using 500-MHz flash-ADC waveform digitizers [12]. Sets of thin trigger counters ("T" and "T" in Fig. 2) surrounding the drift chamber defined the fiducial region, and thin counters surrounding the RS suppressed the muons from $K^+ \to \mu^+ \nu$ and $K^+ \to \mu^+ \nu \gamma$ decays. The trigger requirement of a sufficient time delay (≥ 1.5 ns) between the Čerenkov and I counter signals ensured that the kaons had decayed at rest in the target. A hermetic calorimeter system surrounded the central region; the photons from $K^+ \to \pi^+ \gamma \gamma$ were detected in a lead/scintillator sandwich barrel detector (BV) surrounding the RS, while two endcap calorimeters and other detectors were used for detecting extra particles including photons.

A solenoid surrounding the BV and endcaps provided a 1 T magnetic field along the beam line.

The beam and apparatus in E949 were improved in comparison with those used in E787 [13] for the $K^+ \to \pi^+ \gamma \gamma$ study [2] performed in 1991. The kaon beam line [14], which incorporated two stages of particle separation, reduced the pion contamination. The target, central drift chamber, and RS tracking chambers were replaced by a new target consisting of 0.5-cm square fibers, a new low-mass drift chamber [15], and straw-tuber chambers, respectively. One third of the RS scintillation counters were replaced to increase the light output. A new photon detector, named barrel veto liner (BVL), was installed to added 2.3 radiation lengths of lead/scintillator sandwich material to the BV. The endcaps were replaced by new detectors consisting of undoped-CsI crystals [16], and both the target and endcaps were read out using 500-MHz CCD waveform digitizers [17]. Additional ancillary photon veto systems [18] and an LED flasher system to aid in the RS energy calibration were also introduced.

The new data were acquired in 2002, and the total exposure of kaons entering the target available for the $K^+ \to \pi^+ \gamma \gamma$ study in E949 was $N_K = 1.2 \times 10^{12}$. The trigger required a kaon decay at rest, followed by a π^+ track which came to rest in the RS and coincident activity due to electromagnetic shower in both the BVL and BV, and no extra particles in the endcap or RS counters. The RS counter where the π^+ track came to rest, called the "stopping counter", was requested to be in the 15th or 16th layer in order to suppress the π^+ tracks from the $K^+ \to \pi^+ \pi^0$ decay $(K_{\pi 2})$ with P = 205 MeV/c, R = 30.4 cm, and E = 108 MeV. An improved trigger system [19] in E949, including a programmable trigger board, allowed more efficient running and reduced deadtimes. A total of 1.1×10^7 events met the trigger requirements.

The signature of $K^+ \to \pi^+ \gamma \gamma$ was a kaon decay at rest with a π^+ track in the RS in the kinematic region above the $K_{\pi 2}$ monochromatic peak and photons reconstructed in the BVL and BV detectors. The accepted region of π^+ was 213 MeV/c < P < 234 MeV/c, 33.5 cm < R < 41.3 cm, and 116 MeV < E < 135 MeV; the lower limits corresponded to 3.3, 2.3, and 2.6 standard-deviations above the $K_{\pi 2}$ peak, respectively, in the E949 apparatus. Due to the improvements in the kinematic reconstruction, the region was enlarged from P > 215 MeV/c in E787 [2] and a constrained kinematic fit for consistency with $K^+ \to \pi^+ \gamma \gamma$ was unused in the offline analysis. The timing and energy (E_{γ}) of the photons were determined by grouping

adjacent hit modules in BV and BVL to identify isolated photon showers ("clusters"). The hit position in each module along the beam axis (z) was calculated from the end-to-end time and energy differences; the azimuthal angle (ϕ) of the hit position was determined up to the segmentation of the modules. The location of the photon shower in z and ϕ was obtained by an energy-weighted average of the hit positions and was used, in conjunction with the kaon-decay vertex position in the target, to determine the polar and azimuthal angles of the photon to the π^+ track $(\theta_{\pi^+\gamma}$ and $\phi_{\pi^+\gamma})$. Since the opening angle between two photons from $K^+ \to \pi^+ \gamma \gamma$ gets narrower for the events whose π^+ momentum is close the kinematic end point, the two photons of about one half of the signal events form a single cluster in BVL and BV within their limited position-resolutions; the events with one or two clusters were therefore accepted in the offline analysis. The highest-energy cluster should satisfy 50 MeV $< E_{\gamma} < 320$ MeV, $\theta_{\pi^+\gamma} > 155^{\circ}$, and $\phi_{\pi^+\gamma} > 155^{\circ}$. The energy of the lower-energy cluster, if exists, should be at least 10 MeV. These conditions enabled us to search for the $K^+ \to \pi^+ \gamma$ decay, which is forbidden by angular-momentum conservation and by gauge invariance but is allowed by non-commutative standard model [20], with the same data.

Background sources are: <1> the $K_{\pi 2}$ decay due to mismeasurements of the π^+ and the two photons, including detection in efficiency of the softer of the photons from π^0 if exactly one cluster was reconstructed, $\langle 2 \rangle$ the $K_{\pi 2}$ decay due to the disappearance of the softer photon through overlap with the π^+ track in the RS, <3> the kaon decays with a muon and some photons in the final state, such as $K^+ \to \mu^+ \nu \gamma$ and $K^+ \to \pi^0 \mu^+ \nu$ decays, as well as the $K_{\pi 2}$ decay whose π^+ decays in flight to μ^+ in the detector. These backgrounds were studied from the data by establishing two independent sets of offline selection criteria ("cuts") for each. One set consisted of the cuts on the accepted region of the π^+ momentum, range and kinetic energy. The other set consisted of the cuts on the π^0 invariant-mass (to reject events with $m_{\gamma\gamma} > 100 \text{ MeV}/c^2$), on the extra photon (to reject events with extra activity not associated with the π^+ and the candidate signal photons, identified in each subsystem as hits in the counters in coincidence with the π^+ track within a few ns and with energy above a threshold of typically $\sim 1 \text{ MeV}$), and on the photon cluster [21] (to reject events whose maximum discrepancy among the z-position measurements in the modules of the cluster [22]is larger than 113 cm) to <1>, the cuts on the overlapping photon in the RS (to reject events with a RS counter in which the measured energy was larger than expected from the reconstructed range in that counter [22]) to < 2 >, and the cuts on the $\pi^+ \to \mu^+ \to e^+$ decay sequence, recorded in the RS stopping counter, to < 3 >. The background source of < 4 > the $K_{\pi 2}$ decay in flight in the beam line before the kaon came to rest was studied from the data by the cuts on the time delay (≥ 2 ns) between the π^+ time and the K^+ time measured in the target and on the time difference between the π^+ time in the RS and the K^+ time in the Čerenkov counter. Other backgrounds due to the incident beam particles were found to negligible. In these studies, we inverted at least one of these cuts on the events in order to enhance the background collected by the trigger as well as to prevent candidate events from being examined before the background studies were completed. Possibilities of a correlation between two sets of cuts and a biased estimate of the effectiveness of the cuts, the level of signal acceptance as a function of cut severity, the check of the observed background levels near but outside the signal region in comparison with the predicted background rate, etc. were studied [23] by following the procedures developed through the $K^+ \to \pi^+ \nu \bar{\nu}$ analysis in E949 and E787 [11, 24].

The background levels anticipated with the final analysis cuts were $b_{<1>}=0.017\pm0.006$, $b_{<2>}=0.065\pm0.065$, $b_{<3>}=0.090\pm0.020$, $b_{<4>}=0.025\pm0.014$. In total, $b=0.197\pm0.070$ background events were expected in the signal region. The acceptance (A) and the single event sensitivity (SES) for $K^+ \to \pi^+ \gamma \gamma$: $A=(2.99\pm0.07)\times 10^{-4}$ and $SES=3.72\times 10^{-9}$ at $\hat{c}=1.8$ with the unitarity corrections and $A=(1.18\pm0.04)\times 10^{-4}$ and $SES=9.40\times 10^{-9}$ at $\hat{c}=1.6$ without the corrections, were derived from the acceptance factors in Table I, the total kaon exposure $N_K=1.2\times 10^{12}$, and the K^+ stop efficiency (the fraction of kaons entering the target that came to rest), which was measured with the $K_{\pi 2}$ events collected by the same trigger, of 0.754 ± 0.024 . The former sensitivity was below the predicted branching ratio, 6.10×10^{-9} . In order to verify that the acceptance calculations were correct, the branching ratio for $K^+ \to \mu^+ \nu$ decays was measured with the data sample accumulated by a calibration trigger and 0.628 ± 0.020 , which is consistent with the value in [6], was obtained; this indicates the systematic uncertainty in this study is small.

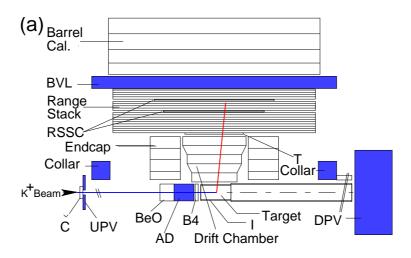
After imposing all analysis cuts, no events were observed in the signal region (Fig. 3). The group of 74 events around R=32 cm, and E=110 MeV [25] was due to the $K_{\pi 2}$ background. In the absence of background and taking 2.44 events instead of zero according to the unified approach [6, 26], we set a 90% confidence level (C.L.) upper limit 9.1×10^{-9} on the branching ratio for $K^+ \to \pi^+ \gamma \gamma$ at $\hat{c}=1.8$ with the unitarity corrections and 2.3×10^{-8} at $\hat{c}=1.6$ without the corrections. The systematic uncertainty was not taken into consideration in

deriving the limit. For the purpose of comparison with the previous E787 results, the 90% C.L. upper limit for the total $K^+ \to \pi^+ \gamma \gamma$ branching ratio assuming the phase-space kinematic distribution was calculated; the limit 6.6×10^{-8} is 7.6 times better than the limit in [2].

The data described above set a 90% C.L. upper limit on the branching ratio for $K^+ \to \pi^+ \gamma$. The previous limit from the E787 study performed in 1996 and 1997 was 3.6 × 10^{-7} [6, 22], and the new limit is 2.5×10^{-9} . The data also set limits on sequential decays in the form $K^+ \to \pi^+ X^0$, $X^0 \to \gamma \gamma$, where X^0 is any short-lived neutral particle with a mass m_{X^0} smaller than 100 MeV/c decaying into two photons. Fig. 4 shows the limit as a function of m_{X^0} for different lifetimes.

The results from this study cannot confirm the ChPT including the unitarity corrections, but the obtained upper limits are the tightest ones that have been experimentally achieved on $K^+ \to \pi^+ \gamma \gamma$, $K^+ \to \pi^+ \gamma$, and $K^+ \to \pi^+ X^0$ decays. The analysis, which was limited by the total exposure of kaons available at the data taking in 2002, has proved that the experiment with kaon decays at rest is suitable to study $K^+ \to \pi^+ \gamma \gamma$ in the π^+ momentum region close to the end point with negligible background contributions. The possibility to observe signal events of $K^+ \to \pi^+ \gamma \gamma$ in the kinematic region, if the ChPT including the unitarity corrections is correct, gives further impetus for additional measurements.

We gratefully acknowledge the dedicated effort of the technical staff supporting E949 and of the Brookhaven AGS Department. This research was supported in part by the U.S. Department of Energy, the Ministry of Education, Culture, Sports, Science and Technology of Japan through the Japan-U.S. Cooperative Research Program in High Energy Physics and under Grant-in-Aids for Scientific Research, the Natural Sciences and Engineering Research Council and the National Research Council of Canada, the Russian Federation State Scientific Center Institute for High Energy Physics, and the Ministry of Industry, Science and New Technologies of the Russian Federation.



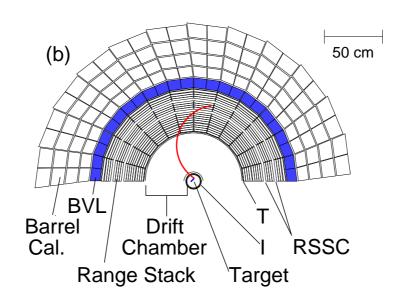


FIG. 2: Schematic side (a) and end (b) views of the upper half of the E949 detector. Č: Čerenkov counter; B4: energy-loss counters; I and T: inner and outer trigger scintillation counters; RSSC: RS straw-tube tracking chambers. New or upgraded subsystems for E949 (shaded) included the barrel veto liner (BVL), collar, upstream photon veto (UPV), active degrader (AD), and downstream photon veto (DPV). In addition, the outer trigger detectors and other trigger and electronics systems were improved in E949.

Acceptance factors	UC	w/o UC	samples
Trigger (in 10^{-1})	0.346	0.061	MC
π^+ reconstruction cuts	0.996	0.996	$K_{\mu 2}$
π^+ fiducial cuts	0.898	0.626	MC
π^+ stop without nuclear interaction	0.492	0.578	MC
or decay-in-flight			
Waveform digitizer $(\pi^+ \to \mu^+ \nu)$ cuts	0.349	0.349	π_{scat}
γ reconstruction and fiducial cuts	0.530	0.492	$MC, K_{\pi 2},$
Extra photon veto cuts	0.216	0.173	$MC, K_{\pi 2},$
π^+ kinematic cuts	0.537	0.537	$K_{\pi 2},\pi_{scat}$
Other cuts on beam and target	0.507	0.507	$K_{\mu 2}$
Trigger-counter efficiency	0.936	0.936	K_B
correction factor	1.93	7.15	MC
Total acceptance (in 10^{-4})	2.99	1.18	

TABLE I: Acceptance factors for $K^+ \to \pi^+ \gamma \gamma$ at $\hat{c} = 1.8$ with the unitarity corrections ("UC") and at $\hat{c} = 1.6$ without the corrections ("w/o UC"), and the samples used to determine them. " $K_{\pi 2}$ ", " $K_{\mu 2}$ ", " π_{scat} ", and " K_B " mean the data samples of $K_{\pi 2}$ decays, $K^+ \to \mu^+ \nu$ decays, scattered beam pions, and kaons entering the target, respectively, which were accumulated by calibration triggers simultaneous to the collection of signal candidates. "MC" means the sample generated by Monte Carlo simulation; the simulation generated the $K^+ \to \pi^+ \gamma \gamma$ decay in P > 200 MeV/c, and the correction factor to convert the acceptance to be in P > 213 MeV/c, by assuming the π^+ spectrum shape in each case, is multiplied.

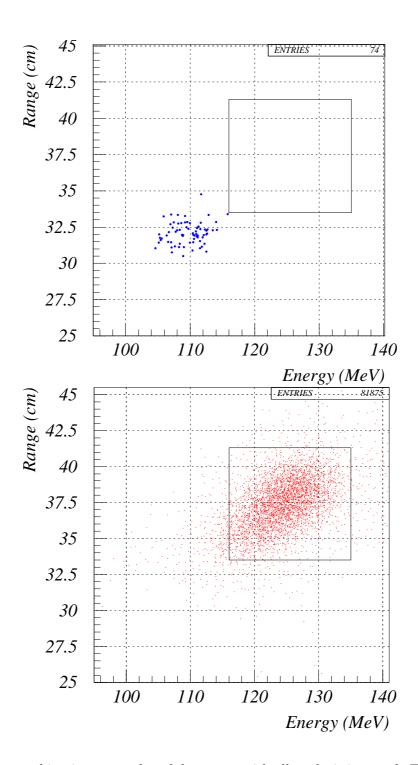


FIG. 3: (a) Range vs. kinetic energy plot of the events with all analysis imposed. The box indicates the signal region for $K^+ \to \pi^+ \gamma \gamma$. (b) The simulated distribution of events from $K^+ \to \pi^+ \gamma \gamma$ decay.

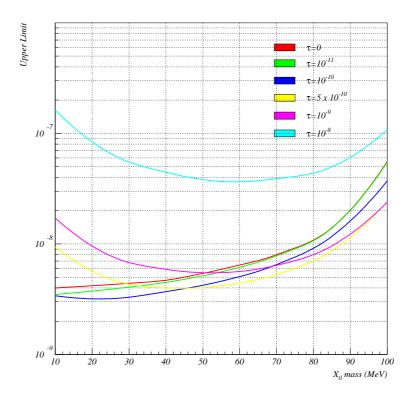


FIG. 4: The 90% C.L. upper limits for the branching ratio of $K^+ \to \pi^+ X^0$, $X^0 \to \gamma \gamma$ for different X^0 lifetimes (τ) in seconds as a function of X^0 mass.

- * Also at the Department of Physics, University of Delhi, Delhi 1100007, India; Present address: Department of Physics and Astronomy, University of Victoria, Victoria, British Columbia, Canada V8W 3P6.
- [†] Present address: Institute for High Energy Physics, Protvino, Moscow Region, 142 280, Russia.
- [‡] Deceased.
- [1] B. Bassalleck *et al.*, E949 Proposal, BNL-67247, TRI-PP-00-06 (1999), [http://www.phy.bnl.gov/e949/] .
- [2] P. Kitching et al., Phys. Rev. Lett. **79**, 4079 (1997).
- [3] J.F. Donoghue, E. Golowich, and B.R. Holstein, *Dynamics of the Standard Model* (Cambridge University Press, Cambridge, 1992), and references therein.
- [4] G. Ecker, A. Pich, and E. de Rafael, Phys. Lett. B189, 363 (1987); Nucl. Phys. B303, 665 (1988); L. Cappiello and G. D'Ambrosio, Nuovo Cimento A99, 155 (1988).
- [5] G. D'Ambrosio and J. Portolés, Phys. Lett. **B389**, 770 (1996); Nucl. Phys. **B492**, 417 (1997).
- [6] Particle Data Group, S. Eidelman et al., Phys. Lett. **B592**, 1 (2004).
- [7] E.g., G. Ecker, A. Pich, and E. de Rafael, Phys. Lett. B237, 481 (1990); A.G. Cohen,
 G. Ecker, and A. Pich, Phys. Lett. B304, 347 (1993).
- [8] A. Alavi-Harati et al., Phys. Rev. Lett. 83, 917 (1999).
- [9] A. Lai et al., Phys. Lett. **B536**, 229 (2002).
- [10] F. Gabbiani and G. Valencia, Phys. Rev. **D64**, 094008 (2001); Phys. Rev. **D66**, 074006 (2002).
- [11] V.V. Anisimovsky et al., Phys. Rev. Lett. 93, 031801 (2004).
- [12] M.S. Atiya et al., Nucl. Instrum. Methods Phys. Res., Sect. A 279, 180 (1989).
- [13] M.S. Atiya et al., Nucl. Instrum. Methods Phys. Res., Sect. A 321, 129 (1992).
- [14] J. Doornbos et al., Nucl. Instrum. Methods Phys. Res., Sect. A 444, 546 (2000).
- [15] E.W. Blackmore et al., Nucl. Instrum. Methods Phys. Res., Sect. A 404, 295 (1998).
- [16] I-H. Chiang et al., IEEE Trans. Nucl. Sci. 42, 394 (1995); T.K. Komatsubara et al.,
 Nucl. Instrum. Methods Phys. Res., Sect. A 404, 315 (1998).
- [17] D.A. Bryman et al., Nucl. Instrum. Methods Phys. Res., Sect A 396, 394 (1997).
- [18] O. Mineev et al., Nucl. Instrum. methods Phys. Res., Sect. A 494, 362 (2002).
- [19] T. Yoshioka et al., IEEE Trans. Nucl. Sci. 51, 334 (2004).

- [20] E.g., J. Trampetić, hep-ph/0212309.
- [21] If two photons from a π^0 hit the same or adjacent modules in BVL and BV, they form a single cluster opposite the π^+ track. Due to the kinematics of $K_{\pi 2}$ and subsequent $\pi^0 \to \gamma \gamma$ decays, the two photons must hit the modules at different z positions along the beam axis.
- [22] S. Adler et al., Phys. Rev. **D65**, 052009 (2002).
- [23] T. Yoshioka, Ph.D. thesis, University of Tokyo, 2005.
- [24] S. Adler et al., Phys. Rev. Lett. 88, 041803 (2002); S. Adler et al., Phys. Rev. Lett.
 84, 3768 (2000); S. Adler et al., Phys. Rev. Lett. 79, 2204 (1997).
- [25] These values are slightly larger than those expected for the $K_{\pi 2}$ decay, due to the fact that the stopping-layer requirement in the trigger would collect the $K_{\pi 2}$ events whose range and energy are measured to be larger in the RS.
- [26] G.J. Feldman and R.D. Cousins, Phys. Rev. **D57**, 3873 (1998).